Name: …….. Date ……..

GAYAZA HIGH SCHOOL

SOLUTIONS TO S.2 MATH WORKSHEET FOUR ON SETS

PREREQUISITE KNOWLEDGE:

- NUMBERS
- INTERGERS

SETS PART I

In modern mathematics, just about everything rests on the very important concept of the **set**.
A set is just a collection of elements, or members. For instance, you could have a set of friends:
F = {Moses, Kenny, Edward, William, Elsam} or a set of numbers: Y = {−3.4, 12, 9999}.

In this worksheet on sets we will solve 12 different types of questions. The questions on sets are basically related on;

- Elements of set and notation of a set.
- Representation of a set.
- Cardinal number of a set.
- Types and pairs of set.

1. Which of the following are sets? Justify your answer.

(a) The collection of all the days in a week beginning with the letter 'T' **Answer: Is a set because it a well-defined collection of distinct objects.**	(b) The collection of all difficult questions in the chapter on sets. **Answer: Is not a set because it is not a well-defined collection of distinct objects.**
(c) The collection of girls in your class. **Answer: Is a set because it a well-defined collection of distinct objects.**	(d)The collection of all rivers in India. **Answer: Is a set because it a well-defined collection of distinct objects.**
(e) The collection of all active teachers in the school. **Answer: Is not a set because it is not a well-defined collection of distinct objects.**	(f) The collection of all integers more than -3. **Answer: Is a set because it a well-defined collection of distinct objects.**

RECALL:
If x is an element of a set A, we write $x \in A$, and if x is not an element of A we write $x \notin A$.
So, using the sets defined above,
$-\mathbf{862} \in \boldsymbol{Z}$, since −862 is an integer, and $\mathbf{2.9} \notin \boldsymbol{M}$, since 2.9 is not greater than 33 .

2. If,
$A = \{3, 5, 7, 9\}$
$B = \{2, 4, 6, 8, 10\}$
$C = \{12, 14, 18, 20, 24\}$
$D = \{21, 26, 31, 36\}$

(a) State whether true or false.

(i) 13 ∈ C **False**	(ii) 6 ∉ A **True**	(iii) 9 ∉ A **False**	(iv) 24 ∈ C **True**
(v) 31 ∈ D **True**	(vi) 36 ∉ D **False**	(vii) 20 ∉ C **False**	(viii) 9 ∈ A **True**

(b) Fill in the blanks.

(i) 3 ∈ A	(ii) 4 ∈ B	(iii) 26 ∉ C	(iv) 8 ∈ B
(v) 5 ⊆ A	(vi) 6 ∉ C	(vii) 21 ⊆ D	(viii) 18 ∉ B

RECALL:
There are two methods of representing a set:
(i) **Roster or tabular form**
(ii) **Set-builder form.**

Roster or tabular form: In roster form, all the elements of a set are listed, the elements are being separated by commas and are enclosed within braces { }.
For Example: Z = the set of all integers = {…,−3,−2,−1,0,1,2,3,…}

Set-builder form: In the set builder form, all the elements of the set, must possess a single property to become the member of that set.
For Example: Z = $\{x: x \text{ is an integer}\}$

You can read Z = $\{x: x \text{ is an integer}\}$ as "The set Z equals all the values of x such that x is an integer."
M = $\{x: x > 3\}$
(This last notation means "all real numbers x such that x is greater than 3." So, for example, 3.1 is in the set M, but 2 is not. The colon ":" means "such that".)

3. Write the following sets in the roster form.

(a) A = The set of all even numbers less than 12 **{2, 4, 6, 8, 10}**	(b) B = The set of all prime numbers greater than 1 but less than 29 **{2, 3, 5, 7, 11, 13, 17, 19, 23}**
(c) C = The set of integers lying between -2 and 2 **{-1, 0, 2}**	(d) D = The set of letters in the word LOYAL **{L, O, Y, A}**
(e) E = The set of vowels in the word CHOICE **{O, I, E}**	(f) F = The set of all factors of 36 **{1, 2, 3, 4, 6, 9, 12, 18, 36}**
(g) G = {x : x ∈ N, 5 < x < 12} **{6, 7, 8, 9, 10, 11}**	(h) H = {x : x is a multiple of 3 and x < 21} **{3, 6, 9, 12, 15, 18}**
(i) I = {x : x is perfect cube 27 < x < 216} **{64, 125}**	(j) J = {x : x = 5n - 3,n ∈ W, and n < 3} **{-3, 2, 7}**
(k) M = {x : x is a positive integer and $x^2 < 40$} **{1, 2, 3, 4, 5, 6}**	(l) N = {x : x is a positive integer and is a divisor of 18} **{1, 2, 3, 6, 9, 18}**
(m) P = {x : x is an integer and x + 1 = 1} **{O}**	(n) Q = {x : x is a color in the rainbow} **{red, orange, yellow, green, blue, indigo, violet}**

4. Write each of the following in set builder form.

(a) A = {5, 10, 15, 20} {x : x is a multiple of 5 and 5 ≤ x ≤ 20}	(b) B = {l, 2, 3, 6, 9, 18} {x : x is a factor of 18}	(c) C = {P, R, I, N, C, A, L} {x : x is a letter of the word 'Principal'}	(d) D = {0} {x : x ∈ W and x < 1}
(e) E = { } {x : x ∈ N and x < 1)	(f) F = {0, 1, 2, 3, ..., 19} {x : x ∈ W and 0 ≤ x ≤ 19}	(g) G = {— **8**, — **6**, — **4**, — **2**} {x : x = -2n and n ∈ N and 1 ≤ n ≤ 4}	(h) H = {Jan, June, July} {x : x is a month of the year beginning with J}
(i) I = {a, e, i, 0, u} {x : x is a vowel of the English alphabet}	(j) J = {a, b, c, d, ..., z} {x : x is a letter of the English alphabet}	(k) $K = \{\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}\}$ {x : x = 1/x, n ∈ N and 1 ≤ n ≤ 6}	(l) L = {1, 3, 5, 7, 9} {x : x is odd, x ≤ 9}

RECALL

Cardinal number of a set.

The **number** of distinct elements in a finite **set** is called its **cardinal number**. It is denoted as n(A) and read as 'the **number** of elements of the **set**'.

For example: **Set** A = {2, 4, 5, 9, 15} has 5 elements, i.e. $n(A) = 5$

5. Write the cardinal number for each of the following.

(a) X = The set of months in a year $\boldsymbol{n(X) = 12}$	(b) Y = The set of letters in the word INTELLIGENT $\boldsymbol{n(Y) = 6}$
(c) Z = The set of prime numbers from 2 to 11 $\boldsymbol{n(Z) = 5}$	(d) P = {x : x is an even prime number} $\boldsymbol{n(P) = 1}$
(e) Q = {x : x is a quadrilateral having 5 sides} $\boldsymbol{n(Q) = 0}$	(f) R = {x : x ∈ I, -5 < x < 2} $\boldsymbol{n(R) = 6}$
(g) S = {x | x ∈ W, x + 2 < 9} $\boldsymbol{n(S) = 7}$	(h) T = {x | x is a prime number which is a divisor of 60} $\boldsymbol{n(T) = 3}$
(i) V = {x : x is a 2-digit number such that the sum of digits is 6} $\boldsymbol{n(V) = 6}$	(j) W = The set of hours in a day $\boldsymbol{n(W) = 24}$

RECALL

If a **set** has a starting and end point both then it is **finite** but if it does not have a starting or end point, then it is **infinite set**.

If a **set** has a **limited number of elements, then** it is **finite** but if **its number of elements is unlimited** then it is **infinite**.

6. Classify the following as finite and infinite sets.

(a) A = {x : x ∈ N and x is even} **Infinite**	(b) B = {x : x ∈ N and x is composite} **Infinite**
(c) C = {x : x ∈ N and 3x -2 = 0} **Finite**	(d) D = {x : x ∈ N and $x^2 = 9$} **Finite**
(e) E = {The set of numbers which are multiples of 3} **Infinite**	(f) G = {The set of letters in the English alphabet} **Finite**
(g) H = {The set of persons living in a house} **Finite**	(h) I = {x : x ∈ I, x < -2} **Finite**
(i) J = {x : x ∈ P, p is a prime number} **Infinite**	(j) K = The set of fractions with numerator 3. **Infinite**

RECALL

You can also have a set which has no elements at all. This special set is called **the empty set or null set**, and we write it with the special symbol ∅ or { }.

A **singleton**, also known as a **unit set**, is a **set** with exactly one element. For example, the **set** {null} is a **singleton** containing the element null.

7. Identify the following as null set or singleton set.

(a) A = {x | x ∈ N, 1 < x < 2} **Null**	(b) P = {Point of intersection of two lines} **Singleton**
(c) C = {x : x is an even prime number greater than 2} **Null**	(d) Q = {x | x is an even prime number} **Singleton**
(e) E = {x : x2 = 9, x is even} **Null**	(f) R = {The set of whole numbers lying between 0 and 2} **Singleton**
(g) B = {0} **Singleton**	(h) D = {The set of largest 1 digit number} **Singleton**
(i) F = {The set of triangles having 4 sides} **Null**	(j) H = {The set of even numbers not divisible by 2} **Null**

RECALL
Equal sets have the exact same elements in them, even though they could be out of order. **Equivalent sets** have different elements but have the same amount of elements. Therefore, if two **sets** have the same cardinality, they are **equivalent.**

8. From the sets given below, select the equal sets.
A = {3, 5, 9, 13}, B = {2, 3, 4, 5}, C = {5, 9, 13, 15}, D = {4, 2, 5, 3}, E = {-2, 2}, F = {o, b}, G = {2, -2}, H = {1, 2}

B = D and E = G

9. Which two sets A and B are equal? Give reasons to support your answer.
(a) A = {x : x is a letter in the word SEAT} and B = {x : x is a letter in the word TASTE}

(b) A = {2, 6, 10, 14} and B = {6, 2, 14, 16}

(c) A = {1, 3, 5, 7, 9} and B = {x : x is a positive odd integer $x \leq 9$}

(d) A = {0} and B = {x : $x > 15$ and $x < 5$}

(a) and (c) Because they have the same elements.

10. Which of the following pairs of sets are equivalent or equal?
(a) A = {x : $x \in N, x \leq 6$} and B = {x : $x \in W, 1 \leq x \leq 6$}

(b) P = {The set of letters in the word 'plane'} and Q = {The set of letters in the word 'plain'}

(c) X = {The set of colors in the rainbow) and Y = {The set of days in a week}

(d) M = {4, 8, 12, 16} and N = {8, 12, 4, 16}

(e) A = {x : $x \in N, x \leq 5$} and B = {x : $x \in I, 5 < x \leq 10$}

Equal sets:	**(a) and (d)**
Equivalent sets:	**(b), (c) and (e)**

RECALL
Two **sets** A and B are said to be **disjoint**, if they do not have any element in common.
Overlapping sets: Two **sets** A and B are said to be **overlapping** if they contain at least one element in common.

11. Find which of the following sets are disjoint or overlapping.
(a) A = {The set of boys in the school} and B = {The set of girls in the school}
(b) P = {The set of letters in the English alphabet} and Q = {The set of vowels in the English alphabet}

(c) X = {x : x is an odd number, $x < 9$} and Y = {x : x is an even number, $x < 10$}

(d) M = {x : x is a factor of 24} and N = {x : x is a multiple of 3, less than 30}

(e) E = {9, 99, 999} and F = {1, 10, 100}

(f) G = {The set of letters in the word TAME} and H = {The set of letters in the word MATE}

Disjoint sets:	**(a), (c) and (e)**
Overlapping sets:	**(b), (d) and (f)**

12. State whether the following are true or false.

(a) The set of letters in the word MASTER is finite. **True**	(b) The set of vowels in the word PLANET is an empty set. **True**
(c) {0} represents a null set. **False**	(d) Equivalent sets are always equal. **False**
(e) An empty set is a finite set. **False**	(f) 10 $\notin$ set of multiples of 5. **True**
(g) $4 \in \{x: 4 \leq x \leq 10\}$ **False**	(h) If two sets are equal, they are also equivalent. **True**

THE END.